Approximation and Entropy Numbers of Volterra Operators with Application to Brownian Motion

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We consider the Volterra integral operator from $L_p(0, \infty) \to L_q(0, \infty)$, defined by

$$(T_{\rho, \psi} f)(s) = \rho(s) \int_0^s \psi(t) f(t) dt$$

and investigate its degree of compactness (entropy numbers and approximation numbers).

The entropy estimates are applied to investigate the small ball behaviour of weighted Wiener processes $\rho W$ in the $L_q$-norm. For example, if $\rho$ satisfies some weak monotonicity conditions at zero and infinity, then

$$\lim_{\epsilon \to 0} \epsilon^2 \log P(\|\rho W\|_q \leq \epsilon) = -k_q \|\rho\|^{2q}_{\frac{2q}{2q+q}}.$$