Asymptotic Behavior of Small Ball Probabilities

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The typical problem of the small ball probabilities is to investigate the asymptotic behavior of $P\{|X| < r\}, \ r \to 0$, for a random vector $X$ taking values in a normed space. As a generic example, take a sample path of a random function as $X$ and consider

$$P\{\sup_{t \in T}|X_t| < r\}, \ r \to 0.$$  

The subject is known for long time to be hard but there was substantial progress during last years. We survey the recent results with special focus on the fruitful connection between Gaussian small ball probabilities and analytical problems of approximation theory.